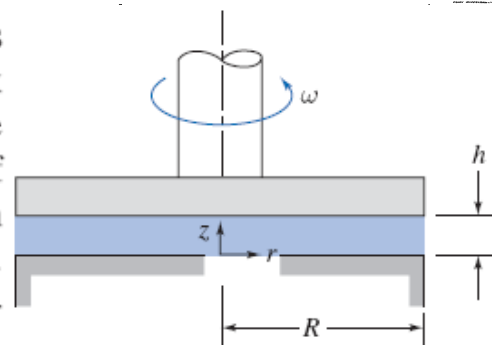


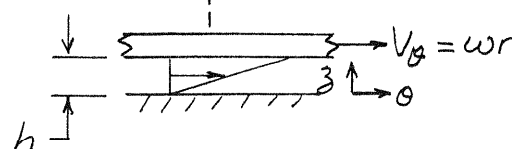
2.66 A proposal has been made to use a pair of parallel disks to measure the viscosity of a liquid sample. The upper disk rotates at height h above the lower disk. The viscosity of the liquid in the gap is to be calculated from measurements of the torque needed to turn the upper disk steadily. Obtain an algebraic expression for the torque needed to turn the disk. Could we use this device to measure the viscosity of a non-Newtonian fluid? Explain.



Solution: Use r, θ, z coordinates at right:

Basic equations: $\tau_{z\theta} = \mu \frac{dv_\theta}{dz}$

$$dT = r dF = r \tau_{z\theta} dA$$



- Assumptions: (1) Newtonian fluid
(2) No-slip condition
(3) Linear velocity profile (in narrow gap)

The velocity at any radial location on the rotating disk is $v_\theta = \omega r$.

Since the velocity profile is linear, then

$$\tau_{z\theta} = \mu \frac{dv_\theta}{dz} = \mu \frac{\Delta v}{\Delta z} = \mu \frac{(\omega r - 0)}{(h - 0)} = \frac{\mu \omega r}{h}$$

and

$$dT = r \tau_{z\theta} dA = r \mu \frac{\omega r}{h} 2\pi r dr = \frac{2\pi \mu \omega r^3}{h} dr$$

Integrating

$$T = \int_A dT = \int_0^R \frac{2\pi \mu \omega r^3}{h} dr = \left[\frac{\pi \mu \omega r^4}{2h} \right]_0^R$$

$$T = \frac{\pi \mu \omega R^4}{2h}$$

The device could not be used to measure the viscosity of a non-Newtonian fluid because the applied shear stress is not uniform. It varies from zero at the center of the disks to $\mu \omega R/h$ at the edge.